Knights and Knaves Logic Puzzles

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1   Knights and Knaves

You are a tourist on an island with some very strange locals. The locals of this island are either knights or knaves. A knight will answer every question truthfully, while a knave will always lie. Other than that, knights and knaves behave the same, so you cannot tell what a person is without hearing what they say.

1.1   Introductory Problems

1. Suppose that you ask a knave the same question two separate times, and that person gives you two different answers. Is this possible?

2. If you ask a knight the same question twice, is it possible for them to give you two different answers?

3. Suppose that you ask a knight and a knave the same question, and both give you the same answer. Is this possible?

4. You meet two locals, Alice and Bernard.

   You ask Alice, "Are you a knight?" Alice doesn't tell you her answer directly but instead whispers it to Bernard.

   You then ask Bernard, "What did Alice say?"

   Bernard says, "She said yes, but she's clearly lying."

   Can you determine if Alice is a knight or knave? What about Bernard?

1.2   Advanced Problems

5. Jerry and Kerry are islanders. You ask Jerry what kind of islanders he and Kerry are, and Jerry responds, "we are both knaves." Can you identify what kinds of islanders Jerry and Kerry are?

6. Larry and Mary are islanders.

   Larry says, "Me and Gary are the same kind." (Kind refers to knight/knave)

   Mary says, "No, we are actually of different kinds."

   What is Larry, and what is Mary?

7. You are lost in the woods and there is a fork in the road ahead of you. There are two islanders at the fork, and one of them is a knight and the other is a knave. Apart from that, you don't know which person is which, and you don't know the one path that will get you out of the woods. You may ask one yes-no question to either one of the people to help you. What do you ask?

8. You meet two more islanders Billy and Bob.

   You ask Billy, "Is Bob a knave?" And like before, Billy whispers his answer to Bob.

   You ask Bob, "Did Billy say 'yes'?"

Bob replies, "No."

Which of the following must be true?

   (a) Billy is a knight.

   (b) Billy is a knave.

   (c) Bob is a knight.

   (d) Bob is a knave.

9. Three inhabitants Albert, Barney, Charlie meet some day, and Albert says either "I am a knight" or "I am a knave", but he said it in a foreign language, so we don't know which phrase he said. We asked Barney what Albert said, and he says "Albert said, 'I am a knave'". After that, Barney says "Charlie is a knave". Then, Charlie says "Albert is a knight". What are Albert, Barney, and Charlie, and why?

10. Four people, Brian, Chris, LeRoy, and Mike live together on this island, and they make the following statements.

Brian: "Mike and I are not the same kind."

Chris: "LeRoy is a knave."

LeRoy: "Chris is a knave."

Mike: "Of the four of us, at least two are knights."

How many of these people are knaves?

1.3 Challenge Problems

11. You meet 10 islanders, who we will call #1 through #10.

#1 says, "At least 1 of us is a knave."

#2 says, "At least 2 of us are knaves."

#3 says, "At least 3 of us are knaves."

$\vdots$

#10 says, "At least 10 of us are knaves."

Which of these people are knaves, and which are knights?

12. You learn that there is a special type of local on this island: togglers. A toggler will either tell the truth or lie the first time you ask it a question. Every question after that, it will "toggle" between telling the truth or lying (e.g. if he told the truth the first time, he will lie the next).

You meet five people, and you know that four of them are togglers and one of them is a knight. You may ask a total of two questions, meaning you can either ask one person two questions or ask two of the people one question each. How can you design a system that guarantees that you will find the knight?